



# Pairwise Comparisons with Flexible Time-Dynamics

## Introduction

**Goal:** predict outcomes of sports games.

**Idea:** associate a (latent) score parameter to each player / team. These parameters drive the outcome probabilities

$$P[i \succ j] = \frac{1}{1 + \exp[-(s_i - s_j)]}$$

"i wins over j" score difference

**Problem:** as players / teams improve and decline over time, their latent score should change.

## Elo System

Update the score after each observation using a stochastic gradient step.

$$s_i^{(k+1)} = s_i^{(k)} + \lambda(\mathbf{1}_{\{i \succ j\}} - P[i \succ j])$$

gradient of log-likelihood

## KickScore

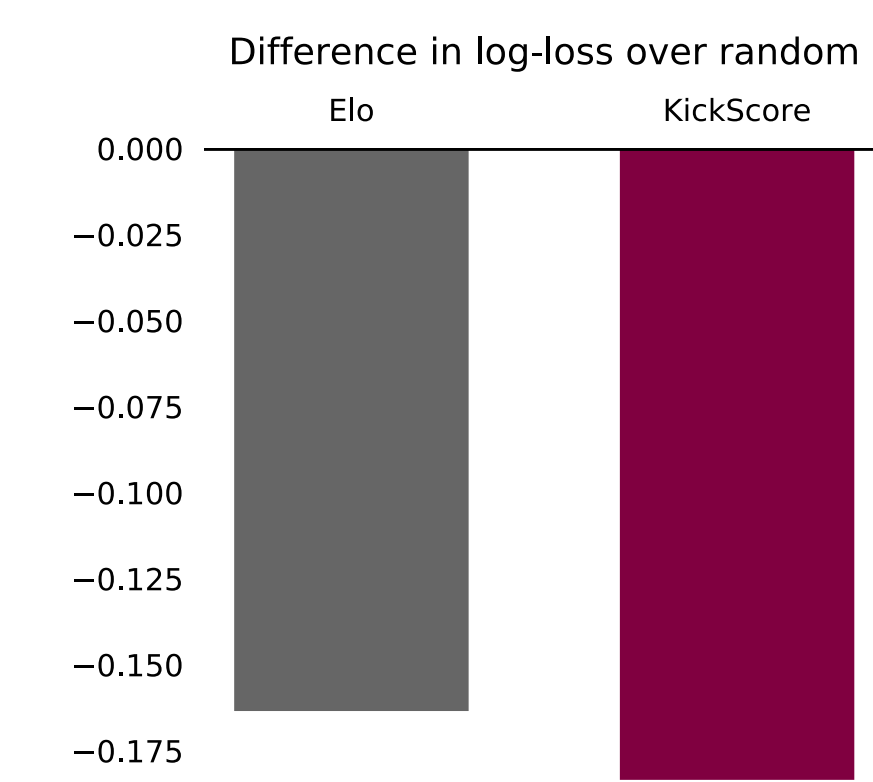
Model the score as a stochastic process.

$$s_i(t) \sim \text{GP}[m(t), k(t, t')]$$

covariance function, encodes dynamics

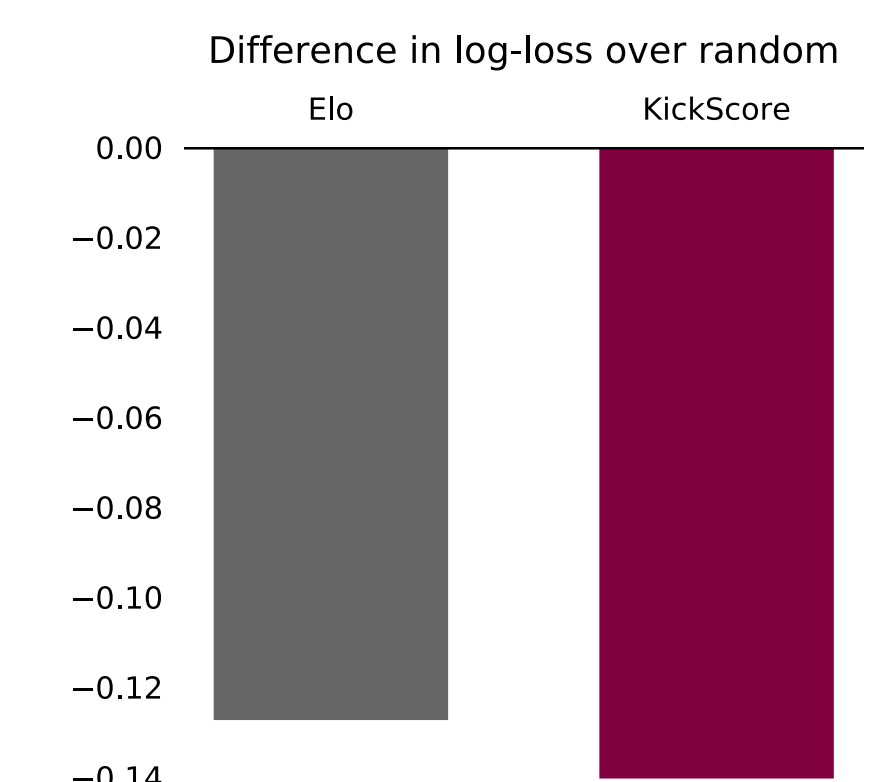
Use approximate Bayesian inference to compute a posterior distribution given the data.

## Predictive Performance



### Football dataset

- 16,091 games  
- 2001 — today  
  
KickScore improves over Elo system by 12%.



### Tennis dataset

- 231,148 games  
- 2010 — today  
  
KickScore improves over Elo system by 10%.

## Inference algorithm

### Variational approximation

$$p(\mathbf{s}_1, \dots, \mathbf{s}_M | \mathcal{D}) \propto \prod_{i=1}^M p(\mathbf{s}_i) \prod_{(i,j,t) \in \mathcal{D}} p(i \succ j | t)$$

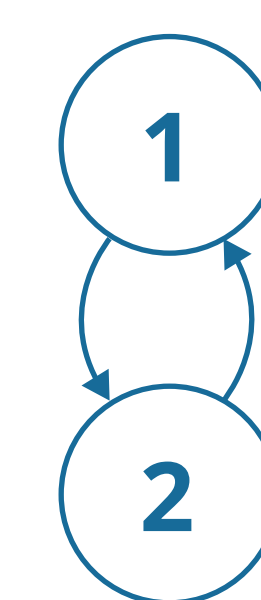
$$\approx q(\mathbf{s}_1, \dots, \mathbf{s}_M) \doteq \prod_i \mathcal{N}(\mathbf{s}_i | \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$$

### Alternative viewpoint

$$q(\mathbf{s}_i) \propto p(\mathbf{s}_i) \prod_{t \in \mathcal{D}} \mathcal{N}[s_i(t) | \tilde{\mu}_{it}, \tilde{\sigma}_{it}]$$

Gaussian "pseudo-observation"

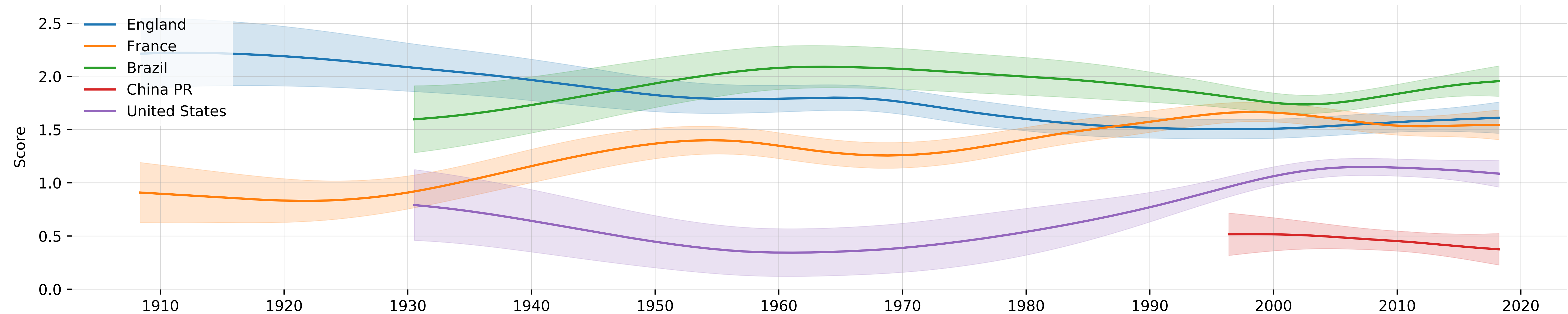
### Linear-time algorithm



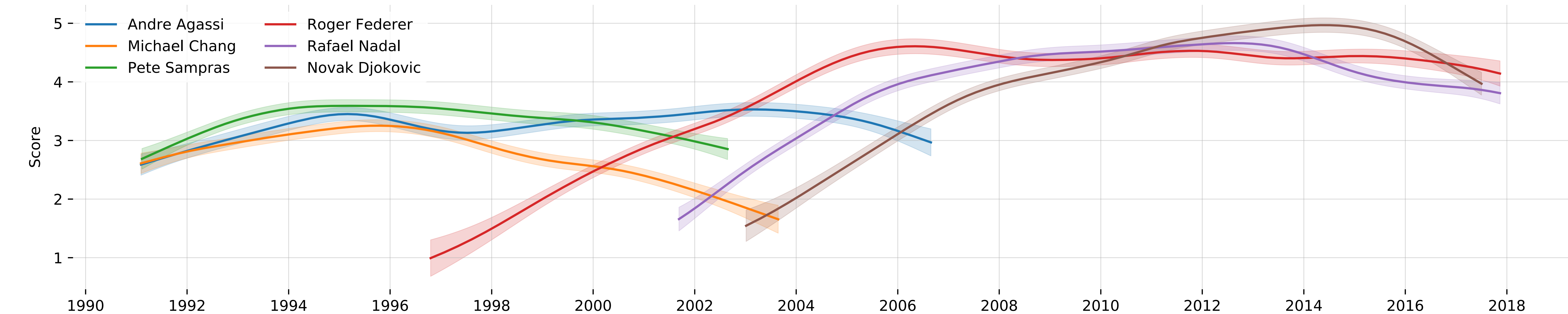
1. Approximate  $p(i \succ j | t)$  by  $\mathcal{N}[s_i(t) | \tilde{\mu}_{it}, \tilde{\sigma}_{it}] \cdot \mathcal{N}[s_j(t) | \tilde{\mu}_{jt}, \tilde{\sigma}_{jt}]$

2. Recompute posterior skill  $q(\mathbf{s}_i) \propto p(\mathbf{s}_i) \prod_{t \in \mathcal{D}} \mathcal{N}[s_i(t) | \tilde{\mu}_{it}, \tilde{\sigma}_{it}]$

## Association football (1908-2018)



## ATP tennis (1991-2017)



## NBA basketball (1946-2018)

